## Exercise 4.1

**Question 1:**

Evaluate the determinants in Exercises 1 and 2.

$$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$$

Answer

$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$ = 2(−1) − 4(−5) = − 2 + 20 = 18

**Question 2:**

Evaluate the determinants in Exercises 1 and 2.

(i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ (ii) $\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$

Answer

(i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ = $(\cos\theta)(\cos\theta) - (-\sin\theta)(\sin\theta) = \cos^2\theta + \sin^2\theta = 1$

(ii) $\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$

$= (x^2 - x + 1)(x + 1) - (x - 1)(x + 1)$

$= x^3 - x^2 + x + x^2 - x + 1 - (x^2 - 1)$

$= x^3 + 1 - x^2 + 1$

$= x^3 - x^2 + 2$

**Question 3:**

If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$

Answer

The given matrix is $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$.

$$\therefore 2A = 2\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$$

$$\therefore \text{L.H.S.} = |2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix} = 2\times4 - 4\times8 = 8 - 32 = -24$$

$$\text{Now, } |A| = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix} = 1\times2 - 2\times4 = 2 - 8 = -6$$

$$\therefore \text{R.H.S.} = 4|A| = 4\times(-6) = -24$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

**Question 4:**

If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that $|3A| = 27|A|$.

Answer

The given matrix is $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$.

It can be observed that in the first column, two entries are zero. Thus, we expand along the first column ($C_1$) for easier calculation.

$$|A| = 1\begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ 0 & 4 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} = 1(4-0) - 0 + 0 = 4$$

$$\therefore 27|A| = 27(4) = 108 \qquad \text{...(i)}$$

$$\text{Now, } 3A = 3\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$$

$$\therefore |3A| = 3\begin{vmatrix} 3 & 6 \\ 0 & 12 \end{vmatrix} - 0\begin{vmatrix} 0 & 3 \\ 0 & 12 \end{vmatrix} + 0\begin{vmatrix} 0 & 3 \\ 3 & 6 \end{vmatrix}$$

$$= 3(36-0) = 3(36) = 108 \qquad \text{...(ii)}$$

From equations (i) and (ii), we have:

$$|3A| = 27|A|$$

Hence, the given result is proved.

**Question 5:**

Evaluate the determinants

(i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ (iii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

(ii) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$ (iv) $\begin{bmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}$

Answer

(i) Let $A = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$.

It can be observed that in the second row, two entries are zero. Thus, we expand along the second row for easier calculation.

$$|A| = -0\begin{vmatrix} -1 & -2 \\ -5 & 0 \end{vmatrix} + 0\begin{vmatrix} 3 & -2 \\ 3 & 0 \end{vmatrix} - (-1)\begin{vmatrix} 3 & -1 \\ 3 & -5 \end{vmatrix} = (-15+3) = -12$$

(ii) Let $A = \begin{bmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{bmatrix}$.

By expanding along the first row, we have:

$$\begin{aligned} |A| &= 3\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} + 4\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + 5\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} \\ &= 3(1+6) + 4(1+4) + 5(3-2) \\ &= 3(7) + 4(5) + 5(1) \\ &= 21 + 20 + 5 = 46 \end{aligned}$$

(iii) Let $A=\begin{bmatrix}0 & 1 & 2\\ -1 & 0 & -3\\ -2 & 3 & 0\end{bmatrix}$.

By expanding along the first row, we have:

$$\begin{aligned}|A| &= 0\begin{vmatrix}0 & -3\\ 3 & 0\end{vmatrix} - 1\begin{vmatrix}-1 & -3\\ -2 & 0\end{vmatrix} + 2\begin{vmatrix}-1 & 0\\ -2 & 3\end{vmatrix}\\ &= 0-1(0-6)+2(-3-0)\\ &= -1(-6)+2(-3)\\ &= 6-6=0\end{aligned}$$

(iv) Let $A=\begin{bmatrix}2 & -1 & -2\\ 0 & 2 & -1\\ 3 & -5 & 0\end{bmatrix}$.

By expanding along the first column, we have:

$$\begin{aligned}|A| &= 2\begin{vmatrix}2 & -1\\ -5 & 0\end{vmatrix} - 0\begin{vmatrix}-1 & -2\\ -5 & 0\end{vmatrix} + 3\begin{vmatrix}-1 & -2\\ 2 & -1\end{vmatrix}\\ &= 2(0-5)-0+3(1+4)\\ &= -10+15=5\end{aligned}$$

**Question 6:**

If $A=\begin{bmatrix}1 & 1 & -2\\ 2 & 1 & -3\\ 5 & 4 & -9\end{bmatrix}$, find $|\mathrm{A}|$.

Answer

Let $A=\begin{bmatrix}1 & 1 & -2\\ 2 & 1 & -3\\ 5 & 4 & -9\end{bmatrix}$.

By expanding along the first row, we have:

$$|A| = 1\begin{vmatrix}1 & -3\\4 & -9\end{vmatrix} - 1\begin{vmatrix}2 & -3\\5 & -9\end{vmatrix} - 2\begin{vmatrix}2 & 1\\5 & 4\end{vmatrix}$$

$$= 1(-9+12) - 1(-18+15) - 2(8-5)$$

$$= 1(3) - 1(-3) - 2(3)$$

$$= 3 + 3 - 6$$

$$= 6 - 6$$

$$= 0$$

**Question 7:**

Find values of $x$, if

(i) $\begin{vmatrix}2 & 4\\2 & 1\end{vmatrix} = \begin{vmatrix}2x & 4\\6 & x\end{vmatrix}$ (ii) $\begin{vmatrix}2 & 3\\4 & 5\end{vmatrix} = \begin{vmatrix}x & 3\\2x & 5\end{vmatrix}$

Answer

(i) $\begin{vmatrix}2 & 4\\5 & 1\end{vmatrix} = \begin{vmatrix}2x & 4\\6 & x\end{vmatrix}$

$$\Rightarrow 2\times 1 - 5\times 4 = 2x\times x - 6\times 4$$

$$\Rightarrow 2 - 20 = 2x^2 - 24$$

$$\Rightarrow 2x^2 = 6$$

$$\Rightarrow x^2 = 3$$

$$\Rightarrow x = \pm\sqrt{3}$$

(ii) $\begin{vmatrix}2 & 3\\4 & 5\end{vmatrix} = \begin{vmatrix}x & 3\\2x & 5\end{vmatrix}$

$$\Rightarrow 2\times 5 - 3\times 4 = x\times 5 - 3\times 2x$$

$$\Rightarrow 10 - 12 = 5x - 6x$$

$$\Rightarrow -2 = -x$$

$$\Rightarrow x = 2$$

**Question 8:**

If $\begin{vmatrix}x & 2\\18 & x\end{vmatrix} = \begin{vmatrix}6 & 2\\18 & 6\end{vmatrix}$, then $x$ is equal to

(A) 6 (B) ±6 (C) −6 (D) 0

Answer

**Answer: B**

$$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$$

$\Rightarrow x^2 - 36 = 36 - 36$

$\Rightarrow x^2 - 36 = 0$

$\Rightarrow x^2 = 36$

$\Rightarrow x = \pm 6$

Hence, the correct answer is B.