Exercise 13.5

Question 1:

A die is thrown 6 times. If 'getting an odd number' is a success, what is the probability of

(i) 5 successes? (ii) at least 5 successes?

(iii) at most 5 successes?

Answer

The repeated tosses of a die are Bernoulli trials. Let X denote the number of successes of getting odd numbers in an experiment of 6 trials.

Probability of getting an odd number in a single throw of a die is, $p=\frac{3}{6}=\frac{1}{2}$

$\therefore q=1-p=\frac{1}{2}$

X has a binomial distribution.

Therefore, P (X = x) = ${}^{n}C_{n-x}q^{n-x}p^{x}$, where $n=0, 1, 2 \ldots n$

$={}^{6}C_{x}\left(\frac{1}{2}\right)^{6-x}\cdot\left(\frac{1}{2}\right)^{x}$

$={}^{6}C_{x}\left(\frac{1}{2}\right)^{6}$

(i) P (5 successes) = P (X = 5)

$={}^{6}C_{5}\left(\frac{1}{2}\right)^{6}$

$=6\cdot\frac{1}{64}$

$=\frac{3}{32}$

(ii) P(at least 5 successes) = P(X ≥ 5)

$= P(X = 5) + P(X = 6)$

$= {}^6C_5\left(\frac{1}{2}\right)^6 + {}^6C_6\left(\frac{1}{2}\right)^6$

$= 6\cdot\frac{1}{64} + 1\cdot\frac{1}{64}$

$= \frac{7}{64}$

(iii) P (at most 5 successes) = P(X ≤ 5)

$= 1 - P(X > 5)$

$= 1 - P(X = 6)$

$= 1 - {}^6C_6\left(\frac{1}{2}\right)^6$

$= 1 - \frac{1}{64}$

$= \frac{63}{64}$

Question 2:

A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two successes.

Answer

The repeated tosses of a pair of dice are Bernoulli trials. Let X denote the number of times of getting doublets in an experiment of throwing two dice simultaneously four times.

Probability of getting doublets in a single throw of the pair of dice is

$p = \frac{6}{36} = \frac{1}{6}$

$\therefore q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$

Clearly, X has the binomial distribution with $n = 4$, $p = \frac{1}{6}$, and $q = \frac{5}{6}$

$\therefore P(X = x) = {}^nC_x q^{n-x} p^x$, where $x = 0, 1, 2, 3 \ldots n$

$$= {}^4C_x \left(\frac{5}{6}\right)^{4-x} \cdot \left(\frac{1}{6}\right)^x$$

$$= {}^4C_x \cdot \frac{5^{4-x}}{6^4}$$

∴ P (2 successes) = P (X = 2)

$$= {}^4C_2 \cdot \frac{5^{4-2}}{6^4}$$

$$= 6 \cdot \frac{25}{1296}$$

$$= \frac{25}{216}$$

Question 3:

There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

Answer

Let X denote the number of defective items in a sample of 10 items drawn successively. Since the drawing is done with replacement, the trials are Bernoulli trials.

$$\Rightarrow p = \frac{5}{100} = \frac{1}{20}$$

$$\therefore q = 1 - \frac{1}{20} = \frac{19}{20}$$

X has a binomial distribution with n = 10 and $p = \frac{1}{20}$

P(X = x) = ${}^nC_x q^{n-x} p^x$, where $x = 0, 1, 2 \dots n$

$$= {}^{10}C_x \left(\frac{19}{20}\right)^{10-x} \cdot \left(\frac{1}{20}\right)^x$$

P (not more than 1 defective item) = P (X ≤ 1)

$= P(X=0)+P(X=1)$

$= {}^{10}C_0\left(\frac{19}{20}\right)^{10}\cdot\left(\frac{1}{20}\right)^{0}+{}^{10}C_1\left(\frac{19}{20}\right)^{9}\cdot\left(\frac{1}{20}\right)^{1}$

$=\left(\frac{19}{20}\right)^{10}+10\left(\frac{19}{20}\right)^{9}\cdot\left(\frac{1}{20}\right)$

$=\left(\frac{19}{20}\right)^{9}\cdot\left[\frac{19}{20}+\frac{10}{20}\right]$

$=\left(\frac{19}{20}\right)^{9}\cdot\left(\frac{29}{20}\right)$

$=\left(\frac{29}{20}\right)\cdot\left(\frac{19}{20}\right)^{9}$

Question 4:

Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that

(i) all the five cards are spades?

(ii) only 3 cards are spades?

(iii) none is a spade?

Answer

Let X represent the number of spade cards among the five cards drawn. Since the drawing of card is with replacement, the trials are Bernoulli trials.

In a well shuffled deck of 52 cards, there are 13 spade cards.

$\Rightarrow p=\frac{13}{52}=\frac{1}{4}$

$\therefore q=1-\frac{1}{4}=\frac{3}{4}$

X has a binomial distribution with n = 5 and $p=\frac{1}{4}$

$P(X=x)={}^{n}C_x q^{n-x}p^{x}$, where $x=0, 1, \ldots n$

$= {}^{5}C_x\left(\frac{3}{4}\right)^{5-x}\left(\frac{1}{4}\right)^{x}$

(i) P (all five cards are spades) = P(X = 5)

$$= {}^5C_5 \left(\frac{3}{4}\right)^0 \cdot \left(\frac{1}{4}\right)^5$$

$$= 1 \cdot \frac{1}{1024}$$

$$= \frac{1}{1024}$$

(ii) P (only 3 cards are spades) = P(X = 3)

$$= {}^5C_3 \cdot \left(\frac{3}{4}\right)^2 \cdot \left(\frac{1}{4}\right)^3$$

$$= 10 \cdot \frac{9}{16} \cdot \frac{1}{64}$$

$$= \frac{45}{512}$$

(iii) P (none is a spade) = P(X = 0)

$$= {}^5C_0 \cdot \left(\frac{3}{4}\right)^5 \cdot \left(\frac{1}{4}\right)^0$$

$$= 1 \cdot \frac{243}{1024}$$

$$= \frac{243}{1024}$$

Question 5:

The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05.

What is the probability that out of 5 such bulbs

(i) none

(ii) not more than one

(iii) more than one

(iv) at least one

will fuse after 150 days of use.

Answer

Let X represent the number of bulbs that will fuse after 150 days of use in an experiment of 5 trials. The trials are Bernoulli trials.

It is given that, $p = 0.05$

$\therefore q = 1 - p = 1 - 0.05 = 0.95$

X has a binomial distribution with *n* = 5 and *p* = 0.05

$\therefore P(X = x) = {}^nC_x q^{n-x} p^x$, where $x = 1, 2, \ldots n$

$= {}^5C_x (0.95)^{5-x} \cdot (0.05)^x$

(i) P (none) = P(X = 0)

$= {}^5C_0 (0.95)^5 \cdot (0.05)^0$

$= 1 \times (0.95)^5$

$= (0.95)^5$

(ii) P (not more than one) = P(X ≤ 1)

$= P(X = 0) + P(X = 1)$

$= {}^5C_0 (0.95)^5 \times (0.05)^0 + {}^5C_1 (0.95)^4 \times (0.05)^1$

$= 1 \times (0.95)^5 + 5 \times (0.95)^4 \times (0.05)$

$= (0.95)^5 + (0.25)(0.95)^4$

$= (0.95)^4 [0.95 + 0.25]$

$= (0.95)^4 \times 1.2$

(iii) P (more than 1) = P(X > 1)

$= 1 - P(X \leq 1)$

$= 1 - P(\text{not more than } 1)$

$= 1 - (0.95)^4 \times 1.2$

(iv) P (at least one) = P(X ≥ 1)

$= 1 - P(X < 1)$

$= 1 - P(X = 0)$

$= 1 - {}^5C_0 (0.95)^5 \times (0.05)^0$

$= 1 - 1 \times (0.95)^5$

$= 1 - (0.95)^5$

Question 6:

A bag consists of 10 balls each marked with one of the digits 0 to 9. If four balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0?

Answer

Let X denote the number of balls marked with the digit 0 among the 4 balls drawn.

Since the balls are drawn with replacement, the trials are Bernoulli trials.

X has a binomial distribution with *n* = 4 and $p=\frac{1}{10}$

$$\therefore q=1-p=1-\frac{1}{10}=\frac{9}{10}$$

$$\therefore P(X=x)={}^{n}C_{x}q^{n-x}\cdot p^{x}, x=1,2,...n$$

$$={}^{4}C_{x}\left(\frac{9}{10}\right)^{4-x}\cdot\left(\frac{1}{10}\right)^{x}$$

P (none marked with 0) = P (X = 0)

$$={}^{4}C_{0}\left(\frac{9}{10}\right)^{4}\cdot\left(\frac{1}{10}\right)^{0}$$

$$=1\cdot\left(\frac{9}{10}\right)^{4}$$

$$=\left(\frac{9}{10}\right)^{4}$$

Question 7:

In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true'; if it falls tails, he answers 'false'. Find the probability that he answers at least 12 questions correctly.

Answer

Let X represent the number of correctly answered questions out of 20 questions.

The repeated tosses of a coin are Bernoulli trails. Since "head" on a coin represents the true answer and "tail" represents the false answer, the correctly answered questions are Bernoulli trials.

$\therefore p = \frac{1}{2}$

$\therefore q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

X has a binomial distribution with n = 20 and $p = \frac{1}{2}$

$\therefore P(X = x) = {}^{n}C_{x} q^{n-x} p^{x}$, where $x = 0, 1, 2, \ldots n$

$$= {}^{20}C_{x} \left(\frac{1}{2}\right)^{20-x} \cdot \left(\frac{1}{2}\right)^{x}$$

$$= {}^{20}C_{x} \left(\frac{1}{2}\right)^{20}$$

P (at least 12 questions answered correctly) = P(X ≥ 12)

$= P(X = 12) + P(X = 13) + \ldots + P(X = 20)$

$= {}^{20}C_{12} \left(\frac{1}{2}\right)^{20} + {}^{20}C_{13} \left(\frac{1}{2}\right)^{20} + \ldots + {}^{20}C_{20} \left(\frac{1}{2}\right)^{20}$

$= \left(\frac{1}{2}\right)^{20} \cdot \left[{}^{20}C_{12} + {}^{20}C_{13} + \ldots + {}^{20}C_{20}\right]$

Question 8:

Suppose X has a binomial distribution $B\left(6, \frac{1}{2}\right)$. Show that X = 3 is the most likely outcome.

(Hint: P(X = 3) is the maximum among all P (x_i), x_i = 0, 1, 2, 3, 4, 5, 6)

Answer

X is the random variable whose binomial distribution is $B\left(6,\frac{1}{2}\right)$.

Therefore, n = 6 and $p=\frac{1}{2}$

$\therefore q=1-p=1-\frac{1}{2}=\frac{1}{2}$

$$\text{Then, } P(X=x)={}^{n}C_{x}q^{n-x}p^{x}$$
$$={}^{6}C_{x}\left(\frac{1}{2}\right)^{6-x}\cdot\left(\frac{1}{2}\right)^{x}$$
$$={}^{6}C_{x}\left(\frac{1}{2}\right)^{6}$$

It can be seen that P(X = x) will be maximum, if ${}^{6}C_{x}$ will be maximum.

$$\text{Then, } {}^{6}C_{0}={}^{6}C_{6}=\frac{6!}{0!\cdot 6!}=1$$
$${}^{6}C_{1}={}^{6}C_{5}=\frac{6!}{1!\cdot 5!}=6$$
$${}^{6}C_{2}={}^{6}C_{4}=\frac{6!}{2!\cdot 4!}=15$$
$${}^{6}C_{3}=\frac{6!}{3!\cdot 3!}=20$$

The value of ${}^{6}C_{3}$ is maximum. Therefore, for x = 3, P(X = x) is maximum.

Thus, X = 3 is the most likely outcome.

Question 9:

On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

Answer

The repeated guessing of correct answers from multiple choice questions are Bernoulli trials. Let X represent the number of correct answers by guessing in the set of 5 multiple choice questions.

Probability of getting a correct answer is, $p = \frac{1}{3}$

$\therefore q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

Clearly, X has a binomial distribution with $n = 5$ and $p = \frac{1}{3}$

$\therefore P(X = x) = {}^{n}C_{x} q^{n-x} p^{x}$

$= {}^{5}C_{x} \left(\frac{2}{3}\right)^{5-x} \cdot \left(\frac{1}{3}\right)^{x}$

P (guessing more than 4 correct answers) = P(X ≥ 4)

$= P(X = 4) + P(X = 5)$

$= {}^{5}C_{4} \left(\frac{2}{3}\right) \cdot \left(\frac{1}{3}\right)^{4} + {}^{5}C_{5} \left(\frac{1}{3}\right)^{5}$

$= 5 \cdot \frac{2}{3} \cdot \frac{1}{81} + 1 \cdot \frac{1}{243}$

$= \frac{10}{243} + \frac{1}{243}$

$= \frac{11}{243}$

Question 10:

A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will in a prize (a) at least once (b) exactly once (c) at least twice?

Answer

Let X represent the number of winning prizes in 50 lotteries. The trials are Bernoulli trials.

Clearly, X has a binomial distribution with $n = 50$ and $p = \frac{1}{100}$

$$\therefore q = 1 - p = 1 - \frac{1}{100} = \frac{99}{100}$$

$$\therefore P(X = x) = {}^{n}C_{x} q^{n-x} p^{x} = {}^{50}C_{x}\left(\frac{99}{100}\right)^{50-x} \cdot \left(\frac{1}{100}\right)^{x}$$

(a) P (winning at least once) = P (X ≥ 1)

$$= 1 - P(X < 1)$$

$$= 1 - P(X = 0)$$

$$= 1 - {}^{50}C_{0}\left(\frac{99}{100}\right)^{50}$$

$$= 1 - 1 \cdot \left(\frac{99}{100}\right)^{50}$$

$$= 1 - \left(\frac{99}{100}\right)^{50}$$

(b) P (winning exactly once) = P(X = 1)

$$= {}^{50}C_{1}\left(\frac{99}{100}\right)^{49} \cdot \left(\frac{1}{100}\right)^{1}$$

$$= 50\left(\frac{1}{100}\right)\left(\frac{99}{100}\right)^{49}$$

$$= \frac{1}{2}\left(\frac{99}{100}\right)^{49}$$

(c) P (at least twice) = P(X ≥ 2)

$= 1 - P(X < 2)$

$= 1 - P(X \le 1)$

$= 1 - [P(X = 0) + P(X = 1)]$

$= [1 - P(X = 0)] - P(X = 1)$

$= 1 - \left(\frac{99}{100}\right)^{50} - \frac{1}{2} \cdot \left(\frac{99}{100}\right)^{49}$

$= 1 - \left(\frac{99}{100}\right)^{49} \cdot \left[\frac{99}{100} + \frac{1}{2}\right]$

$= 1 - \left(\frac{99}{100}\right)^{49} \cdot \left(\frac{149}{100}\right)$

$= 1 - \left(\frac{149}{100}\right)\left(\frac{99}{100}\right)^{49}$

Question 11:

Find the probability of getting 5 exactly twice in 7 throws of a die.

Answer

The repeated tossing of a die are Bernoulli trials. Let X represent the number of times of getting 5 in 7 throws of the die.

Probability of getting 5 in a single throw of the die, $p = \frac{1}{6}$

$\therefore q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$

Clearly, X has the probability distribution with n = 7 and $p = \frac{1}{6}$

$\therefore P(X = x) = {}^{n}C_{x} q^{n-x} p^{x} = {}^{7}C_{x} \left(\frac{5}{6}\right)^{7-x} \cdot \left(\frac{1}{6}\right)^{x}$

P (getting 5 exactly twice) = P(X = 2)

$$= {}^7C_2\left(\frac{5}{6}\right)^5\cdot\left(\frac{1}{6}\right)^2$$

$$=21\cdot\left(\frac{5}{6}\right)^5\cdot\frac{1}{36}$$

$$=\left(\frac{7}{12}\right)\left(\frac{5}{6}\right)^5$$

Question 12:

Find the probability of throwing at most 2 sixes in 6 throws of a single die.

Answer

The repeated tossing of the die are Bernoulli trials. Let X represent the number of times of getting sixes in 6 throws of the die.

Probability of getting six in a single throw of die, $p=\frac{1}{6}$

$$\therefore q=1-p=1-\frac{1}{6}=\frac{5}{6}$$

Clearly, X has a binomial distribution with n = 6

$$\therefore P(X=x)={}^nC_x q^{n-x}p^x={}^6C_x\left(\frac{5}{6}\right)^{6-x}\cdot\left(\frac{1}{6}\right)^x$$

P (at most 2 sixes) = P(X ≤ 2)

$$= P(X=0)+P(X=1)+P(X=2)$$

$$= {}^6C_0\left(\frac{5}{6}\right)^6 + {}^6C_1\cdot\left(\frac{5}{6}\right)^5\cdot\left(\frac{1}{6}\right) + {}^6C_2\left(\frac{5^4}{6}\right)\cdot\left(\frac{1}{6}\right)^2$$

$$= 1\cdot\left(\frac{5}{6}\right)^6 + 6\cdot\frac{1}{6}\cdot\left(\frac{5}{6}\right)^5 + 15.\frac{1}{36}\cdot\left(\frac{5}{6}\right)^4$$

$$= \left(\frac{5}{6}\right)^6 + \left(\frac{5}{6}\right)^5 + \frac{5}{12}\cdot\left(\frac{5}{6}\right)^4$$

$$= \left(\frac{5}{6}\right)^4\left[\left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right) + \left(\frac{5}{12}\right)\right]$$

$$= \left(\frac{5}{6}\right)^4\cdot\left[\frac{25}{36}+\frac{5}{6}+\frac{5}{12}\right]$$

$$= \left(\frac{5}{6}\right)^4\cdot\left[\frac{25+30+15}{36}\right]$$

$$= \frac{70}{36}\cdot\left(\frac{5}{6}\right)^4$$

$$= \frac{35}{18}\cdot\left(\frac{5}{6}\right)^4$$

Question 13:

It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles, 9 are defective?

Answer

The repeated selections of articles in a random sample space are Bernoulli trails. Let X denote the number of times of selecting defective articles in a random sample space of 12 articles.

Clearly, X has a binomial distribution with n = 12 and p = 10% $= \frac{10}{100} = \frac{1}{10}$

$$\therefore q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

$$\therefore P(X=x) = {}^nC_x q^{n-x} p^x = {}^{12}C_x\left(\frac{9}{10}\right)^{12-x}\cdot\left(\frac{1}{10}\right)^x$$

P (selecting 9 defective articles) = ${}^{12}C_9\left(\frac{9}{10}\right)^3\left(\frac{1}{10}\right)^9$

$= 220 \cdot \frac{9^3}{10^3} \cdot \frac{1}{10^9}$

$= \frac{22 \times 9^3}{10^{11}}$

Question 14:

In a box containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective is

(A) 10^{-1}

(B) $\left(\frac{1}{2}\right)^5$

(C) $\left(\frac{9}{10}\right)^5$

(D) $\frac{9}{10}$

Answer

The repeated selections of defective bulbs from a box are Bernoulli trials. Let X denote the number of defective bulbs out of a sample of 5 bulbs.

Probability of getting a defective bulb, $p = \frac{10}{100} = \frac{1}{10}$

$\therefore q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$

Clearly, X has a binomial distribution with n = 5 and $p = \frac{1}{10}$

$\therefore P(X = x) = {}^nC_x q^{n-x} p^x = {}^5C_x\left(\frac{9}{10}\right)^{5-x}\left(\frac{1}{10}\right)^x$

P (none of the bulbs is defective) = P(X = 0)

$= {}^5C_0 \cdot \left(\frac{9}{10}\right)^5$

$= 1 \cdot \left(\frac{9}{10}\right)^5$

$= \left(\frac{9}{10}\right)^5$

The correct answer is C.

Question 15:

The probability that a student is not a swimmer is $\frac{1}{5}$. Then the probability that out of five students, four are swimmers is

(A) ${}^5C_4\left(\frac{4}{5}\right)^4\frac{1}{5}$ **(B)** $\left(\frac{4}{5}\right)^4\frac{1}{5}$

(C) ${}^5C_1\frac{1}{5}\left(\frac{4}{5}\right)^4$ **(D)** None of these

Answer

The repeated selection of students who are swimmers are Bernoulli trials. Let X denote the number of students, out of 5 students, who are swimmers.

Probability of students who are not swimmers, $q = \frac{1}{5}$

$\therefore p = 1 - q = 1 - \frac{1}{5} = \frac{4}{5}$

Clearly, X has a binomial distribution with n = 5 and $p = \frac{4}{5}$

$P(X = x) = {}^nC_x q^{n-x} p^x = {}^5C_x \left(\frac{1}{5}\right)^{5-x} \cdot \left(\frac{4}{5}\right)^x$

P (four students are swimmers) = P(X = 4) $= {}^5C_4\left(\frac{1}{5}\right)\cdot\left(\frac{4}{5}\right)^4$

Therefore, the correct answer is A.

Miscellaneous Solutions

Question 1:

A and B are two events such that P (A) ≠ 0. Find P (B|A), if

(i) A is a subset of B (ii) A ∩ B = Φ

Answer

It is given that, P (A) ≠ 0

(i) A is a subset of B.

$\Rightarrow A \cap B = A$

$\therefore P(A \cap B) = P(B \cap A) = P(A)$

$$\therefore P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A)}{P(A)} = 1$$

(ii) $A \cap B = \phi$

$\Rightarrow P(A \cap B) = 0$

$$\therefore P(B|A) = \frac{P(A \cap B)}{P(A)} = 0$$

Question 2:

A couple has two children,

(i) Find the probability that both children are males, if it is known that at least one of the children is male.

(ii) Find the probability that both children are females, if it is known that the elder child is a female.

Answer

If a couple has two children, then the sample space is

S = {(b, b), (b, g), (g, b), (g, g)}

(i) Let E and F respectively denote the events that both children are males and at least one of the children is a male.

$$\therefore E \cap F = \{(b, b)\} \Rightarrow P(E \cap F) = \frac{1}{4}$$

$$P(E) = \frac{1}{4}$$

$$P(F) = \frac{3}{4}$$

$$\Rightarrow P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

(ii) Let A and B respectively denote the events that both children are females and the elder child is a female.

$$A = \{(g, g)\} \Rightarrow P(A) = \frac{1}{4}$$

$$B = \{(g, b), (g, g)\} \Rightarrow P(B) = \frac{2}{4}$$

$$A \cap B = \{(g, g)\} \Rightarrow P(A \cap B) = \frac{1}{4}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{2}{4}} = \frac{1}{2}$$

Question 3:

Suppose that 5% of men and 0.25% of women have grey hair. A haired person is selected at random. What is the probability of this person being male?

Assume that there are equal number of males and females.

Answer

It is given that 5% of men and 0.25% of women have grey hair.

Therefore, percentage of people with grey hair = (5 + 0.25) % = 5.25%

☐ Probability that the selected haired person is a male $= \frac{5}{5.25} = \frac{20}{21}$

Question 4:

Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed?

Answer

A person can be either right-handed or left-handed.

It is given that 90% of the people are right-handed.

$$\therefore p = \text{P}(\text{right-handed}) = \frac{9}{10}$$

$$q = \text{P}(\text{left-handed}) = 1 - \frac{9}{10} = \frac{1}{10}$$

Using binomial distribution, the probability that more than 6 people are right-handed is given by,

$$\sum_{r=7}^{10} {}^{10}C_r p^r q^{n-r} = \sum_{r=7}^{10} {}^{10}C_r \left(\frac{9}{10}\right)^r \left(\frac{1}{10}\right)^{10-r}$$

Therefore, the probability that at most 6 people are right-handed

= 1 − P (more than 6 are right-handed)

$$= 1 - \sum_{r=7}^{10} {}^{10}C_r (0.9)^r (0.1)^{10-r}$$

Question 5:

An urn contains 25 balls of which 10 balls bear a mark 'X' and the remaining 15 bear a mark 'Y'. A ball is drawn at random from the urn, its mark is noted down and it is replaced. If 6 balls are drawn in this way, find the probability that

(i) all will bear 'X' mark.

(ii) not more than 2 will bear 'Y' mark.

(iii) at least one ball will bear 'Y' mark

(iv) the number of balls with 'X' mark and 'Y' mark will be equal.

Answer

Total number of balls in the urn = 25

Balls bearing mark 'X' = 10

Balls bearing mark 'Y' = 15

p = P (ball bearing mark 'X') $= \frac{10}{25} = \frac{2}{5}$

q = P (ball bearing mark 'Y') $= \frac{15}{25} = \frac{3}{5}$

Six balls are drawn with replacement. Therefore, the number of trials are Bernoulli trials.

Let Z be the random variable that represents the number of balls with 'Y' mark on them in the trials.

Clearly, Z has a binomial distribution with n = 6 and $p = \frac{2}{5}$.

☐ P (Z = z) = ${}^{n}C_{z} p^{n-z} q^{z}$

(i) P (all will bear 'X' mark) = P (Z = 0) = ${}^{6}C_{0}\left(\frac{2}{5}\right)^{6} = \left(\frac{2}{5}\right)^{6}$

(ii) P (not more than 2 bear 'Y' mark) = P (Z ≤ 2)

= P (Z = 0) + P (Z = 1) + P (Z = 2)

$= {}^{6}C_{0}(p)^{6}(q)^{0} + {}^{6}C_{1}(p)^{5}(q)^{1} + {}^{6}C_{2}(p)^{4}(q)^{2}$

$= \left(\frac{2}{5}\right)^{6} + 6\left(\frac{2}{5}\right)^{5}\left(\frac{3}{5}\right) + 15\left(\frac{2}{5}\right)^{4}\left(\frac{3}{5}\right)^{2}$

$= \left(\frac{2}{5}\right)^{4}\left[\left(\frac{2}{5}\right)^{2} + 6\left(\frac{2}{5}\right)\left(\frac{3}{5}\right) + 15\left(\frac{3}{5}\right)^{2}\right]$

$= \left(\frac{2}{5}\right)^{4}\left[\frac{4}{25} + \frac{36}{25} + \frac{135}{25}\right]$

$= \left(\frac{2}{5}\right)^{4}\left[\frac{175}{25}\right]$

$= 7\left(\frac{2}{5}\right)^{4}$

(iii) P (at least one ball bears 'Y' mark) = P (Z ≥ 1) = 1 − P (Z = 0)

$= 1 - \left(\frac{2}{5}\right)^{6}$

(iv) P (equal number of balls with 'X' mark and 'Y' mark) = P (Z = 3)

$= {}^6C_3\left(\frac{2}{54}\right)^3\left(\frac{3}{5}\right)^3$

$= \frac{20\times8\times27}{15625}$

$= \frac{864}{3125}$

Question 6:

In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles?

Answer

Let *p* and *q* respectively be the probabilities that the player will clear and knock down the hurdle.

$\therefore p = \frac{5}{6}$

$\Rightarrow q = 1 - p = 1 - \frac{5}{6} = \frac{1}{6}$

Let X be the random variable that represents the number of times the player will knock down the hurdle.

Therefore, by binomial distribution, we obtain

P (X = *x*) = ${}^nC_x p^{n-x} q^x$

P (player knocking down less than 2 hurdles) = P (X < 2)

= P (X = 0) + P (X = 1)

= ${}^{10}C_0(q)^0(p)^{10} + {}^{10}C_1(q)(p)^9$

$$=\left(\frac{5}{6}\right)^{10}+10\cdot\frac{1}{6}\cdot\left(\frac{5}{6}\right)^{9}$$

$$=\left(\frac{5}{6}\right)^{9}\left[\frac{5}{6}+\frac{10}{6}\right]$$

$$=\frac{5}{2}\left(\frac{5}{6}\right)^{9}$$

$$=\frac{(5)^{10}}{2\times(6)^{9}}$$

Question 7:

A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

Answer

The probability of getting a six in a throw of die is $\frac{1}{6}$ and not getting a six is $\frac{5}{6}$.

Let $p=\frac{1}{6}$ and $q=\frac{5}{6}$

The probability that the 2 sixes come in the first five throws of the die is

$${}^{5}C_{2}\left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right)^{3}=\frac{10\times(5)^{3}}{(6)^{5}}$$

☐ Probability that third six comes in the sixth throw $=\frac{10\times(5)^{3}}{(6)^{5}}\times\frac{1}{6}$

$$=\frac{10\times125}{(6)^{6}}$$

$$=\frac{10\times125}{46656}$$

$$=\frac{625}{23328}$$

Question 8:

If a leap year is selected at random, what is the chance that it will contain 53 Tuesdays?

Answer

In a leap year, there are 366 days i.e., 52 weeks and 2 days.

In 52 weeks, there are 52 Tuesdays.

Therefore, the probability that the leap year will contain 53 Tuesdays is equal to the probability that the remaining 2 days will be Tuesdays.

The remaining 2 days can be

Monday and Tuesday

Tuesday and Wednesday

Wednesday and Thursday

Thursday and Friday

Friday and Saturday

Saturday and Sunday

Sunday and Monday

Total number of cases = 7

Favourable cases = 2

☐Probability that a leap year will have 53 Tuesdays $= \frac{2}{7}$

Question 9:

An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be at least 4 successes.

Answer

The probability of success is twice the probability of failure.

Let the probability of failure be *x*.

☐ Probability of success = 2*x*

$x + 2x = 1$

$\Rightarrow 3x = 1$

$\Rightarrow x = \frac{1}{3}$

$\therefore 2x = \frac{2}{3}$

Let $p = \frac{1}{3}$ and $q = \frac{2}{3}$

Let X be the random variable that represents the number of successes in six trials.

By binomial distribution, we obtain

P (X = x) = ${}^nC_x p^{n-x} q^x$

Probability of at least 4 successes = P (X ≥ 4)

= P (X = 4) + P (X = 5) + P (X = 6)

$$= {}^6C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2 + {}^6C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right) + {}^6C_6\left(\frac{2}{3}\right)^6$$

$$= \frac{15(2)^4}{3^6} + \frac{6(2)^5}{3^6} + \frac{(2)^6}{3^6}$$

$$= \frac{(2)^4}{(3)^6}[15+12+4]$$

$$= \frac{31\times 2^4}{(3)^6}$$

$$= \frac{31}{9}\left(\frac{2}{3}\right)^4$$

Question 10:

How many times must a man toss a fair coin so that the probability of having at least one head is more than 90%?

Answer

Let the man toss the coin n times. The n tosses are n Bernoulli trials.

Probability (p) of getting a head at the toss of a coin is $\frac{1}{2}$.

□ $p = \frac{1}{2}$ □ $q = \frac{1}{2}$

$$\therefore P(X = x) = {}^nC_x p^{n-x} q^x = {}^nC_x\left(\frac{1}{2}\right)^{n-x}\left(\frac{1}{2}\right)^x = {}^nC_x\left(\frac{1}{2}\right)^n$$

It is given that,

P (getting at least one head) > $\frac{90}{100}$

P (x ≥ 1) > 0.9

☐ 1 − P (x = 0) > 0.9

$$1-{}^{n}C_{0}.\frac{1}{2^{n}}>0.9$$

$${}^{n}C_{0}.\frac{1}{2^{n}}<0.1$$

$$\frac{1}{2^{n}}<0.1$$

$$2^{n}>\frac{1}{0.1}$$

$$2^{n}>10 \qquad ...(1)$$

The minimum value of n that satisfies the given inequality is 4.

Thus, the man should toss the coin 4 or more than 4 times.

Question 11:

In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins/loses.

Answer

In a throw of a die, the probability of getting a six is $\frac{1}{6}$ and the probability of not getting a 6 is $\frac{5}{6}$.

Three cases can occur.

i. If he gets a six in the first throw, then the required probability is $\frac{1}{6}$.

Amount he will receive = Re 1

ii. If he does not get a six in the first throw and gets a six in the second throw, then probability $=\left(\frac{5}{6}\times\frac{1}{6}\right)=\frac{5}{36}$

Amount he will receive = −Re 1 + Re 1 = 0

iii. If he does not get a six in the first two throws and gets a six in the third throw,

then probability $= \left(\frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right) = \frac{25}{216}$

Amount he will receive = −Re 1 − Re 1 + Re 1 = −1

Expected value he can win $= \frac{1}{6}(1) + \left(\frac{5}{6} \times \frac{1}{6}\right)(0) + \left[\left(\frac{5}{6}\right)^2 \times \frac{1}{6}\right](-1)$

$$= \frac{1}{6} \pm \frac{25}{216}$$

$$= \frac{36-25}{216} = \frac{11}{216}$$

Question 12:

Suppose we have four boxes. A, B, C and D containing coloured marbles as given below:

Box	**Marble colour**		
	Red	**White**	**Black**
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

One of the boxes has been selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from box A?, box B?, box C?

Answer

Let R be the event of drawing the red marble.

Let E_A, E_B, and E_C respectively denote the events of selecting the box A, B, and C.

Total number of marbles = 40

Number of red marbles = 15

$$\therefore P(R) = \frac{15}{40} = \frac{3}{8}$$

Probability of drawing the red marble from box A is given by P (E_A|R).

$$\therefore P(E_A|R)=\frac{P(E_A\cap R)}{P(R)}=\frac{\frac{1}{40}}{\frac{3}{8}}=\frac{1}{15}$$

Probability that the red marble is from box B is P (E_B|R).

$$\Rightarrow P(E_B|R)=\frac{P(E_B\cap R)}{P(R)}=\frac{\frac{6}{40}}{\frac{3}{8}}=\frac{2}{5}$$

Probability that the red marble is from box C is P (E_C|R).

$$\Rightarrow P(E_C|R)=\frac{P(E_C\cap R)}{P(R)}=\frac{\frac{8}{40}}{\frac{3}{8}}=\frac{8}{15}$$

Question 13:

Assume that the chances of the patient having a heart attack are 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?

Answer

Let A, E_1, and E_2 respectively denote the events that a person has a heart attack, the selected person followed the course of yoga and meditation, and the person adopted the drug prescription.

$$\therefore P(A)=0.40$$

$$P(E_1)=P(E_2)=\frac{1}{2}$$

$$P(A|E_1)=0.40\times 0.70=0.28$$

$$P(A|E_2)=0.40\times 0.75=0.30$$

Probability that the patient suffering a heart attack followed a course of meditation and yoga is given by P (E_1|A).

$$P(E_1|A)=\frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)}$$

$$=\frac{\frac{1}{2}\times 0.28}{\frac{1}{2}\times 0.28+\frac{1}{2}\times 0.30}$$

$$=\frac{14}{29}$$

Question 14:

If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive? (Assume that the individual entries of the determinant are chosen independently, each value being assumed with probability $\frac{1}{2}$).

Answer

The total number of determinants of second order with each element being 0 or 1 is $(2)^4 = 16$

The value of determinant is positive in the following cases. $\begin{vmatrix}1 & 0\\0 & 1\end{vmatrix}, \begin{vmatrix}1 & 1\\0 & 1\end{vmatrix}, \begin{vmatrix}1 & 0\\1 & 1\end{vmatrix}$

☐ Required probability $=\frac{3}{16}$

Question 15:

An electronic assembly consists of two subsystems, say, A and B. From previous testing procedures, the following probabilities are assumed to be known:

P(A fails) = 0.2

P(B fails alone) = 0.15

P(A and B fail) = 0.15

Evaluate the following probabilities

(i) P(A fails| B has failed) (ii) P(A fails alone)

Answer

Let the event in which A fails and B fails be denoted by E_A and E_B.

$P(E_A) = 0.2$

$P(E_A$ □ $E_B) = 0.15$

P (B fails alone) $= P(E_B) - P(E_A$ □ $E_B)$

□ $0.15 = P(E_B) - 0.15$

□ $P(E_B) = 0.3$

(i) $P(E_A|E_B) = \frac{P(E_A \cap E_B)}{P(E_B)} = \frac{0.15}{0.3} = 0.5$

(ii) P (A fails alone) $= P(E_A) - P(E_A$ □ $E_B)$

$= 0.2 - 0.15$

$= 0.05$

Question 16:

Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

Answer

Let E_1 and E_2 respectively denote the events that a red ball is transferred from bag I to II and a black ball is transferred from bag I to II.

$P(E_1) = \frac{3}{7}$ and $P(E_2) = \frac{4}{7}$

Let A be the event that the ball drawn is red.

When a red ball is transferred from bag I to II,

$P(A|E_1) = \frac{5}{10} = \frac{1}{2}$

When a black ball is transferred from bag I to II,

$P(A|E_2) = \frac{4}{10} = \frac{2}{5}$

$$\therefore P(E_2|A) = \frac{P(E_2)P(A|E_2)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$$

$$= \frac{\frac{4}{7} \times \frac{2}{5}}{\frac{3}{7} \times \frac{1}{2} + \frac{4}{7} \times \frac{2}{5}}$$

$$= \frac{16}{31}$$

Question 17:

If A and B are two events such that P (A) ≠ 0 and P(B|A) = 1, then.

(A) A ☐ B

(B) B ☐ A

(C) B = Φ

(D) A = Φ

Answer

P (A) ≠ 0 and $P(B|A) = 1$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$1 = \frac{P(B \cap A)}{P(A)}$$

$$P(A) = P(B \cap A)$$

$$\Rightarrow A \subset B$$

Thus, the correct answer is A.

Question 18:

If P (A|B) > P (A), then which of the following is correct:

(A) P (B|A) < P (B) **(B)** P (A ☐ B) < P (A).P (B)

(C) P (B|A) > P (B) **(D)** P (B|A) = P (B)

Answer

$$P(A|B) > P(A)$$
$$\Rightarrow \frac{P(A \cap B)}{P(B)} > P(A)$$
$$\Rightarrow P(A \cap B) > P(A) \cdot P(B)$$
$$\Rightarrow \frac{P(A \cap B)}{P(A)} > P(B)$$
$$\Rightarrow P(B|A) > P(B)$$

Thus, the correct answer is C.

Question 19:

If A and B are any two events such that P (A) + P (B) − P (A and B) = P (A), then

(A) P (B|A) = 1 **(B)** P (A|B) = 1

(C) P (B|A) = 0 **(D)** P (A|B) = 0

Answer

$$P(A) + P(B) - P(A \text{ and } B) = P(A)$$
$$\Rightarrow P(A) + P(B) - P(A \cap B) = P(A)$$
$$\Rightarrow P(B) - P(A \cap B) = 0$$
$$\Rightarrow P(A \cap B) = P(B)$$
$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$$

Thus, the correct answer is B.