Exercise 7.2

Question 1:

$$\frac{2x}{1+x^2}$$

Answer

Let $1+x^2 = t$

$\therefore 2x\, dx = dt$

$$\Rightarrow \int \frac{2x}{1+x^2}dx = \int \frac{1}{t}dt$$

$$= \log|t| + C$$

$$= \log|1+x^2| + C$$

$$= \log(1+x^2) + C$$

Question 2:

$$\frac{(\log x)^2}{x}$$

Answer

Let $\log |x| = t$

$\therefore \frac{1}{x}dx = dt$

$$\Rightarrow \int \frac{\left(\log|x|\right)^2}{x}dx = \int t^2 dt$$

$$= \frac{t^3}{3} + C$$

$$= \frac{\left(\log|x|\right)^3}{3} + C$$

Question 3:

$$\frac{1}{x + x\log x}$$

Answer

$$\frac{1}{x + x\log x} = \frac{1}{x\left(1 + \log x\right)}$$

Let 1 + log x = t

$$\therefore \frac{1}{x}dx = dt$$

$$\Rightarrow \int \frac{1}{x\left(1+\log x\right)}dx = \int \frac{1}{t}dt$$

$$= \log|t| + C$$

$$= \log|1 + \log x| + C$$

Question 4:

sin x · sin (cos x)

Answer

$\sin x \cdot \sin(\cos x)$

Let $\cos x = t$

$\therefore -\sin x \, dx = dt$

$$\Rightarrow \int \sin x \cdot \sin(\cos x)\,dx = -\int \sin t \, dt$$
$$= -[-\cos t] + C$$
$$= \cos t + C$$
$$= \cos(\cos x) + C$$

Question 5:

$\sin(ax+b)\cos(ax+b)$

Answer

$$\sin(ax+b)\cos(ax+b) = \frac{2\sin(ax+b)\cos(ax+b)}{2} = \frac{\sin 2(ax+b)}{2}$$

Let $2(ax+b) = t$

$\therefore 2a\,dx = dt$

$$\Rightarrow \int \frac{\sin 2(ax+b)}{2}dx = \frac{1}{2}\int \frac{\sin t \ dt}{2a}$$

$$= \frac{1}{4a}[-\cos t] + C$$

$$= \frac{-1}{4a}\cos 2(ax+b) + C$$

Question 6:

$\sqrt{ax+b}$

Answer

Let *ax* + *b* = *t*

⇒ *adx* = *dt*

$$\therefore dx = \frac{1}{a}dt$$

$$\Rightarrow \int (ax+b)^{\frac{1}{2}} dx = \frac{1}{a}\int t^{\frac{1}{2}} dt$$

$$= \frac{1}{a}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{2}{3a}(ax+b)^{\frac{3}{2}} + C$$

Question 7:

$x\sqrt{x+2}$

Answer

Let $(x+2) = t$

$\therefore dx = dt$

$$\Rightarrow \int x\sqrt{x+2}\,dx = \int (t-2)\sqrt{t}\,dt$$
$$= \int \left(t^{\frac{3}{2}} - 2t^{\frac{1}{2}} \right) dt$$
$$= \int t^{\frac{3}{2}} dt - 2\int t^{\frac{1}{2}} dt$$
$$= \frac{t^{\frac{5}{2}}}{\frac{5}{2}} - 2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C$$
$$= \frac{2}{5} t^{\frac{5}{2}} - \frac{4}{3} t^{\frac{3}{2}} + C$$
$$= \frac{2}{5}(x+2)^{\frac{5}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$$

Question 8:

$x\sqrt{1+2x^2}$

Answer

Let $1 + 2x^2 = t$

$\therefore 4x\,dx = dt$

$$\Rightarrow \int x\sqrt{1+2x^2}\,dx = \int \frac{\sqrt{t}\,dt}{4}$$

$$= \frac{1}{4}\int t^{\frac{1}{2}}dt$$

$$= \frac{1}{4}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{1}{6}\left(1+2x^2\right)^{\frac{3}{2}} + C$$

Question 9:

$(4x+2)\sqrt{x^2+x+1}$

Answer

Let $x^2+x+1=t$

∴ $(2x + 1)dx = dt$

$$\int (4x+2)\sqrt{x^2+x+1}\ dx$$

$$= \int 2\sqrt{t}\ dt$$

$$= 2\int \sqrt{t}\ dt$$

$$= 2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$

$$= \frac{4}{3}\left(x^2+x+1\right)^{\frac{3}{2}} + C$$

Question 10:

$$\frac{1}{x-\sqrt{x}}$$

Answer

$$\frac{1}{x-\sqrt{x}}=\frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)}$$

Let $\left(\sqrt{x}-1\right)=t$

$\therefore \frac{1}{2\sqrt{x}}dx=dt$

$$\Rightarrow \int\frac{1}{\sqrt{x}\left(\sqrt{x}-1\right)}dx=\int\frac{2}{t}dt$$

$$=2\log\left|t\right|+\mathrm{C}$$

$$=2\log\left|\sqrt{x}-1\right|+\mathrm{C}$$

Question 11:

$$\frac{x}{\sqrt{x+4}},x>0$$

Answer

Let $x+4=t$

$\therefore$ *dx* = *dt*

$$\int \frac{x}{\sqrt{x+4}}dx = \int \frac{(t-4)}{\sqrt{t}}dt$$

$$= \int\left(\sqrt{t} - \frac{4}{\sqrt{t}}\right)dt$$

$$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 4\left(\frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right) + C$$

$$= \frac{2}{3}(t)^{\frac{3}{2}} - 8(t)^{\frac{1}{2}} + C$$

$$= \frac{2}{3}t \cdot t^{\frac{1}{2}} - 8t^{\frac{1}{2}} + C$$

$$= \frac{2}{3}t^{\frac{1}{2}}(t-12) + C$$

$$= \frac{2}{3}(x+4)^{\frac{1}{2}}(x+4-12) + C$$

$$= \frac{2}{3}\sqrt{x+4}(x-8) + C$$

Question 12:

$(x^3-1)^{\frac{1}{3}}x^5$

Answer

Let $x^3 - 1 = t$

$\therefore\ 3x^2dx = dt$

$$\Rightarrow \int\left(x^3-1\right)^{\frac{1}{3}} x^5 dx = \int\left(x^3-1\right)^{\frac{1}{3}} x^3 \cdot x^2 dx$$

$$= \int t^{\frac{1}{3}}(t+1)\frac{dt}{3}$$

$$= \frac{1}{3}\int\left(t^{\frac{4}{3}}+t^{\frac{1}{3}}\right)dt$$

$$= \frac{1}{3}\left[\frac{t^{\frac{7}{3}}}{\frac{7}{3}}+\frac{t^{\frac{4}{3}}}{\frac{4}{3}}\right]+\mathrm{C}$$

$$= \frac{1}{3}\left[\frac{3}{7}t^{\frac{7}{3}}+\frac{3}{4}t^{\frac{4}{3}}\right]+\mathrm{C}$$

$$= \frac{1}{7}\left(x^3-1\right)^{\frac{7}{3}}+\frac{1}{4}\left(x^3-1\right)^{\frac{4}{3}}+\mathrm{C}$$

Question 13:

$$\frac{x^2}{\left(2+3x^3\right)^3}$$

Answer

Let $2+3x^3 = t$

$\therefore 9x^2\, dx = dt$

$$\Rightarrow \int \frac{x^2}{\left(2+3x^3\right)^3}dx = \frac{1}{9}\int \frac{dt}{(t)^3}$$

$$= \frac{1}{9}\left[\frac{t^{-2}}{-2}\right] + \text{C}$$

$$= \frac{-1}{18}\left(\frac{1}{t^2}\right) + \text{C}$$

$$= \frac{-1}{18\left(2+3x^3\right)^2} + \text{C}$$

Question 14:

$$\frac{1}{x\left(\log x\right)^m}, \; x > 0$$

Answer

Let $\log x = t$

$$\therefore \frac{1}{x}dx = dt$$

$$\Rightarrow \int \frac{1}{x\left(\log x\right)^m}dx = \int \frac{dt}{(t)^m}$$

$$= \left(\frac{t^{-m+1}}{1-m}\right) + \text{C}$$

$$= \frac{\left(\log x\right)^{1-m}}{(1-m)} + \text{C}$$

Question 15:

$$\frac{x}{9-4x^2}$$

Answer

Let $9-4x^2=t$

$\therefore -8x\ dx = dt$

$$\Rightarrow \int \frac{x}{9-4x^2}dx = \frac{-1}{8}\int \frac{1}{t}dt$$
$$= \frac{-1}{8}\log|t| + C$$
$$= \frac{-1}{8}\log\left|9-4x^2\right| + C$$

Question 16:

e^{2x+3}

Answer

Let $2x+3=t$

$\therefore 2dx = dt$

$$\Rightarrow \int e^{2x+3}dx = \frac{1}{2}\int e^t dt$$
$$= \frac{1}{2}\left(e^t\right) + C$$
$$= \frac{1}{2}e^{(2x+3)} + C$$

Question 17:

$$\frac{x}{e^{x^2}}$$

Answer

Let $x^2 = t$

$\therefore 2xdx = dt$

$$\Rightarrow \int \frac{x}{e^{x^2}}dx = \frac{1}{2}\int \frac{1}{e^t}dt$$

$$= \frac{1}{2}\int e^{-t}dt$$

$$= \frac{1}{2}\left(\frac{e^{-t}}{-1}\right) + \text{C}$$

$$= -\frac{1}{2}e^{-x^2} + \text{C}$$

$$= \frac{-1}{2e^{x^2}} + \text{C}$$

Question 18:

$$\frac{e^{\tan^{-1}x}}{1+x^2}$$

Answer

Let $\tan^{-1}x = t$

$\therefore \frac{1}{1+x^2}dx = dt$

$$\Rightarrow \int \frac{e^{\tan^{-1} x}}{1+x^2} dx = \int e^t dt$$

$$= e^t + C$$

$$= e^{\tan^{-1} x} + C$$

Question 19:

$$\frac{e^{2x}-1}{e^{2x}+1}$$

Answer

$$\frac{e^{2x}-1}{e^{2x}+1}$$

Dividing numerator and denominator by e^x, we obtain

$$\frac{\frac{\left(e^{2x}-1\right)}{e^x}}{\frac{\left(e^{2x}+1\right)}{e^x}} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Let $e^x + e^{-x} = t$

$\therefore \left(e^x - e^{-x}\right) dx = dt$

$$\Rightarrow \int \frac{e^{2x}-1}{e^{2x}+1} dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$$

$$= \int \frac{dt}{t}$$

$$= \log|t| + C$$

$$= \log\left|e^x + e^{-x}\right| + C$$

Question 20:

$$\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$$

Answer

Let $e^{2x}+e^{-2x}=t$

$\therefore \left(2e^{2x}-2e^{-2x}\right)dx=dt$

$$\Rightarrow 2\left(e^{2x}-e^{-2x}\right)dx=dt$$

$$\Rightarrow \int\left(\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}\right)dx=\int\frac{dt}{2t}$$

$$=\frac{1}{2}\int\frac{1}{t}dt$$

$$=\frac{1}{2}\log|t|+\text{C}$$

$$=\frac{1}{2}\log\left|e^{2x}+e^{-2x}\right|+\text{C}$$

Question 21:

$$\tan^2(2x-3)$$

Answer

$$\tan^2(2x-3)=\sec^2(2x-3)-1$$

Let $2x - 3 = t$

$\therefore 2dx = dt$

$$\Rightarrow \int \tan^2(2x-3)\,dx = \int\left[\left(\sec^2(2x-3)\right)-1\right]dx$$

$$= \frac{1}{2}\int\left(\sec^2 t\right)dt - \int 1dx$$

$$= \frac{1}{2}\int \sec^2 t\,dt - \int 1dx$$

$$= \frac{1}{2}\tan t - x + C$$

$$= \frac{1}{2}\tan(2x-3) - x + C$$

Question 22:

$\sec^2(7-4x)$

Answer

Let 7 − 4*x* = *t*

∴ −4*dx* = *dt*

$$\therefore \int \sec^2(7-4x)\,dx = \frac{-1}{4}\int \sec^2 t\,dt$$

$$= \frac{-1}{4}(\tan t) + C$$

$$= \frac{-1}{4}\tan(7-4x) + C$$

Question 23:

$\frac{\sin^{-1} x}{\sqrt{1-x^2}}$

Answer

Let $\sin^{-1} x = t$

$\therefore \frac{1}{\sqrt{1-x^2}}dx = dt$

$\Rightarrow \int \frac{\sin^{-1} x}{\sqrt{1-x^2}}dx = \int t\, dt$

$= \frac{t^2}{2} + C$

$= \frac{\left(\sin^{-1} x\right)^2}{2} + C$

Question 24:

$\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}$

Answer

$\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x} = \frac{2\cos x - 3\sin x}{2\left(3\cos x + 2\sin x\right)}$

Let $3\cos x + 2\sin x = t$

$\therefore \left(-3\sin x + 2\cos x\right)dx = dt$

$$\int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}dx = \int \frac{dt}{2t}$$

$$= \frac{1}{2}\int \frac{1}{t}dt$$

$$= \frac{1}{2}\log\left|t\right| + C$$

$$= \frac{1}{2}\log\left|2\sin x + 3\cos x\right| + C$$

Question 25:

$$\frac{1}{\cos^2 x(1-\tan x)^2}$$

Answer

$$\frac{1}{\cos^2 x(1-\tan x)^2}=\frac{\sec^2 x}{(1-\tan x)^2}$$

Let $(1-\tan x)=t$

$\therefore$ $-\sec^2 x dx = dt$

$$\Rightarrow \int\frac{\sec^2 x}{(1-\tan x)^2}dx = \int\frac{-dt}{t^2}$$

$$= -\int t^{-2}dt$$

$$= +\frac{1}{t}+C$$

$$= \frac{1}{(1-\tan x)}+C$$

Question 26:

$$\frac{\cos\sqrt{x}}{\sqrt{x}}$$

Answer

Let $\sqrt{x}=t$

$\therefore$ $\frac{1}{2\sqrt{x}}dx = dt$

$$\Rightarrow \int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx = 2\int \cos t\,dt$$
$$= 2\sin t + C$$
$$= 2\sin\sqrt{x} + C$$

Question 27:

$\sqrt{\sin 2x}\cos 2x$

Answer

Let sin 2x = t

$\therefore$ $2\cos 2x\,dx = dt$

$$\Rightarrow \int \sqrt{\sin 2x}\;\cos 2x\,dx = \frac{1}{2}\int \sqrt{t}\,dt$$
$$= \frac{1}{2}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$$
$$= \frac{1}{3}t^{\frac{3}{2}} + C$$
$$= \frac{1}{3}(\sin 2x)^{\frac{3}{2}} + C$$

Question 28:

$\frac{\cos x}{\sqrt{1+\sin x}}$

Answer

Let $1 + \sin x = t$

$\therefore \cos x\, dx = dt$

$$\Rightarrow \int \frac{\cos x}{\sqrt{1+\sin x}} dx = \int \frac{dt}{\sqrt{t}}$$
$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$
$$= 2\sqrt{t} + C$$
$$= 2\sqrt{1+\sin x} + C$$

Question 29:

cot *x* log sin *x*

Answer

Let log sin *x* = *t*

$$\Rightarrow \frac{1}{\sin x} \cdot \cos x \; dx = dt$$
$$\therefore \cot x \; dx = dt$$

$$\Rightarrow \int \cot x \; \log \sin x \; dx = \int t\, dt$$
$$= \frac{t^2}{2} + C$$
$$= \frac{1}{2}(\log \sin x)^2 + C$$

Question 30:

$$\frac{\sin x}{1+\cos x}$$

Answer

Let 1 + cos *x* = *t*

$\therefore -\sin x\, dx = dt$

$$\Rightarrow \int \frac{\sin x}{1+\cos x}\, dx = \int -\frac{dt}{t}$$
$$= -\log|t| + C$$
$$= -\log|1+\cos x| + C$$

Question 31:

$$\frac{\sin x}{(1+\cos x)^2}$$

Answer

Let $1 + \cos x = t$

$\therefore -\sin x\, dx = dt$

$$\Rightarrow \int \frac{\sin x}{(1+\cos x)^2}\, dx = \int -\frac{dt}{t^2}$$
$$= -\int t^{-2} dt$$
$$= \frac{1}{t} + C$$
$$= \frac{1}{1+\cos x} + C$$

Question 32:

$$\frac{1}{1+\cot x}$$

Answer

$$\text{Let } I = \int \frac{1}{1+\cot x}dx$$
$$= \int \frac{1}{1+\frac{\cos x}{\sin x}}dx$$
$$= \int \frac{\sin x}{\sin x+\cos x}dx$$
$$= \frac{1}{2}\int \frac{2\sin x}{\sin x+\cos x}dx$$
$$= \frac{1}{2}\int \frac{(\sin x+\cos x)+(\sin x-\cos x)}{(\sin x+\cos x)}dx$$
$$= \frac{1}{2}\int 1\,dx+\frac{1}{2}\int \frac{\sin x-\cos x}{\sin x+\cos x}dx$$
$$= \frac{1}{2}(x)+\frac{1}{2}\int \frac{\sin x-\cos x}{\sin x+\cos x}dx$$

Let sin x + cos x = t ⇒ (cos x − sin x) dx = dt

$$\therefore I = \frac{x}{2}+\frac{1}{2}\int \frac{-(dt)}{t}$$
$$= \frac{x}{2}-\frac{1}{2}\log|t|+C$$
$$= \frac{x}{2}-\frac{1}{2}\log|\sin x+\cos x|+C$$

Question 33:

$$\frac{1}{1-\tan x}$$

Answer

$$\text{Let } I = \int \frac{1}{1-\tan x}dx$$
$$= \int \frac{1}{1-\frac{\sin x}{\cos x}}dx$$
$$= \int \frac{\cos x}{\cos x - \sin x}dx$$
$$= \frac{1}{2}\int \frac{2\cos x}{\cos x - \sin x}dx$$
$$= \frac{1}{2}\int \frac{(\cos x - \sin x) + (\cos x + \sin x)}{(\cos x - \sin x)}dx$$
$$= \frac{1}{2}\int 1\,dx + \frac{1}{2}\int \frac{\cos x + \sin x}{\cos x - \sin x}dx$$
$$= \frac{x}{2} + \frac{1}{2}\int \frac{\cos x + \sin x}{\cos x - \sin x}dx$$

Put $\cos x - \sin x = t \Rightarrow (-\sin x - \cos x)\,dx = dt$

$$\therefore I = \frac{x}{2} + \frac{1}{2}\int \frac{-(dt)}{t}$$
$$= \frac{x}{2} - \frac{1}{2}\log|t| + C$$
$$= \frac{x}{2} - \frac{1}{2}\log|\cos x - \sin x| + C$$

Question 34:

$$\frac{\sqrt{\tan x}}{\sin x \cos x}$$

Answer

$$\text{Let } I = \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$$
$$= \int \frac{\sqrt{\tan x} \times \cos x}{\sin x \cos x \times \cos x} dx$$
$$= \int \frac{\sqrt{\tan x}}{\tan x \cos^2 x} dx$$
$$= \int \frac{\sec^2 x \, dx}{\sqrt{\tan x}}$$

$$\text{Let } \tan x = t \Rightarrow \sec^2 x \, dx = dt$$

$$\therefore I = \int \frac{dt}{\sqrt{t}}$$
$$= 2\sqrt{t} + \text{C}$$
$$= 2\sqrt{\tan x} + \text{C}$$

Question 35:

$$\frac{(1+\log x)^2}{x}$$

Answer

Let 1 + log x = t

$$\therefore \frac{1}{x} dx = dt$$

$$\Rightarrow \int \frac{(1+\log x)^2}{x} dx = \int t^2 dt$$
$$= \frac{t^3}{3} + \text{C}$$
$$= \frac{(1+\log x)^3}{3} + \text{C}$$

Question 36:

$$\frac{(x+1)(x+\log x)^2}{x}$$

Answer

$$\frac{(x+1)(x+\log x)^2}{x}=\left(\frac{x+1}{x}\right)(x+\log x)^2=\left(1+\frac{1}{x}\right)(x+\log x)^2$$

Let $(x+\log x)=t$

$\therefore \left(1+\frac{1}{x}\right)dx=dt$

$$\Rightarrow \int\left(1+\frac{1}{x}\right)(x+\log x)^2\,dx=\int t^2 dt$$

$$=\frac{t^3}{3}+C$$

$$=\frac{1}{3}(x+\log x)^3+C$$

Question 37:

$$\frac{x^3\sin\left(\tan^{-1}x^4\right)}{1+x^8}$$

Answer

Let $x^4 = t$

$\therefore 4x^3\,dx = dt$

$$\Rightarrow \int \frac{x^3 \sin\left(\tan^{-1} x^4\right)}{1+x^8} dx = \frac{1}{4} \int \frac{\sin\left(\tan^{-1} t\right)}{1+t^2} dt \qquad ...(1)$$

Let $\tan^{-1} t = u$

$\therefore \frac{1}{1+t^2} dt = du$

From (1), we obtain

$$\int \frac{x^3 \sin\left(\tan^{-1} x^4\right) dx}{1+x^8} = \frac{1}{4} \int \sin u \, du$$

$$= \frac{1}{4}\left(-\cos u\right) + C$$

$$= \frac{-1}{4} \cos\left(\tan^{-1} t\right) + C$$

$$= \frac{-1}{4} \cos\left(\tan^{-1} x^4\right) + C$$

Question 38:

$\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx$ equals

(A) $10^x - x^{10} + C$ (B) $10^x + x^{10} + C$

(C) $\left(10^x - x^{10}\right)^{-1} + C$ (D) $\log\left(10^x + x^{10}\right) + C$

Answer

Let $x^{10} + 10^x = t$

$\therefore \left(10x^9 + 10^x \log_e 10\right) dx = dt$

$$\Rightarrow \int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx = \int \frac{dt}{t}$$

$$= \log t + C$$

$$= \log\left(10^x + x^{10}\right) + C$$

Hence, the correct Answer is D.

Question 39:

$\int \frac{dx}{\sin^2 x \cos^2 x}$ equals

A. $\tan x + \cot x + C$

B. $\tan x - \cot x + C$

C. $\tan x \cot x + C$

D. $\tan x - \cot 2x + C$

Answer

$$\text{Let } I = \int \frac{dx}{\sin^2 x \cos^2 x}$$

$$= \int \frac{1}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} dx + \int \frac{\cos^2 x}{\sin^2 x \cos^2 x} dx$$

$$= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx$$

$$= \tan x - \cot x + C$$

Hence, the correct Answer is B.