

Exercise 7.7

Question 1:

$$\sqrt{4-x^2}$$

Answer

$$\text{Let } I = \int \sqrt{4-x^2} dx = \int \sqrt{(2)^2 - (x)^2} dx$$

$$\text{It is known that, } \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$\begin{aligned} \therefore I &= \frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} + C \\ &= \frac{x}{2} \sqrt{4-x^2} + 2 \sin^{-1} \frac{x}{2} + C \end{aligned}$$

Question 2:

$$\sqrt{1-4x^2}$$

Answer

$$\text{Let } I = \int \sqrt{1-4x^2} dx = \int \sqrt{(1)^2 - (2x)^2} dx$$

$$\text{Let } 2x = t \Rightarrow 2 dx = dt$$

$$\therefore I = \frac{1}{2} \int \sqrt{(1)^2 - (t)^2} dt$$

$$\text{It is known that, } \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$\begin{aligned} \Rightarrow I &= \frac{1}{2} \left[\frac{t}{2} \sqrt{1-t^2} + \frac{1}{2} \sin^{-1} t \right] + C \\ &= \frac{t}{4} \sqrt{1-t^2} + \frac{1}{4} \sin^{-1} t + C \\ &= \frac{2x}{4} \sqrt{1-4x^2} + \frac{1}{4} \sin^{-1} 2x + C \\ &= \frac{x}{2} \sqrt{1-4x^2} + \frac{1}{4} \sin^{-1} 2x + C \end{aligned}$$

Question 3:

$$\sqrt{x^2 + 4x + 6}$$

Answer

$$\begin{aligned} \text{Let } I &= \int \sqrt{x^2 + 4x + 6} \, dx \\ &= \int \sqrt{x^2 + 4x + 4 + 2} \, dx \\ &= \int \sqrt{(x^2 + 4x + 4) + 2} \, dx \\ &= \int \sqrt{(x+2)^2 + (\sqrt{2})^2} \, dx \end{aligned}$$

$$\text{It is known that, } \int \sqrt{x^2 + a^2} \, dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + C$$

$$\begin{aligned} \therefore I &= \frac{(x+2)}{2} \sqrt{x^2 + 4x + 6} + \frac{2}{2} \log |(x+2) + \sqrt{x^2 + 4x + 6}| + C \\ &= \frac{(x+2)}{2} \sqrt{x^2 + 4x + 6} + \log |(x+2) + \sqrt{x^2 + 4x + 6}| + C \end{aligned}$$

Question 4:

$$\sqrt{x^2 + 4x + 1}$$

Answer

$$\begin{aligned} \text{Let } I &= \int \sqrt{x^2 + 4x + 1} \, dx \\ &= \int \sqrt{(x^2 + 4x + 4) - 3} \, dx \\ &= \int \sqrt{(x+2)^2 - (\sqrt{3})^2} \, dx \end{aligned}$$

$$\text{It is known that, } \int \sqrt{x^2 - a^2} \, dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C$$

$$\therefore I = \frac{(x+2)}{2} \sqrt{x^2 + 4x + 1} - \frac{3}{2} \log |(x+2) + \sqrt{x^2 + 4x + 1}| + C$$

Question 5:

$$\sqrt{1-4x-x^2}$$

Answer

$$\begin{aligned}\text{Let } I &= \int \sqrt{1-4x-x^2} \, dx \\ &= \int \sqrt{1-(x^2+4x+4-4)} \, dx \\ &= \int \sqrt{1+4-(x+2)^2} \, dx \\ &= \int \sqrt{(\sqrt{5})^2-(x+2)^2} \, dx\end{aligned}$$

$$\text{It is known that, } \int \sqrt{a^2-x^2} \, dx = \frac{x}{2} \sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$\therefore I = \frac{(x+2)}{2} \sqrt{1-4x-x^2} + \frac{5}{2} \sin^{-1} \left(\frac{x+2}{\sqrt{5}} \right) + C$$

Question 6:

$$\sqrt{x^2+4x-5}$$

Answer

$$\begin{aligned}\text{Let } I &= \int \sqrt{x^2+4x-5} \, dx \\ &= \int \sqrt{(x^2+4x+4)-9} \, dx \\ &= \int \sqrt{(x+2)^2-(3)^2} \, dx\end{aligned}$$

$$\text{It is known that, } \int \sqrt{x^2-a^2} \, dx = \frac{x}{2} \sqrt{x^2-a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2-a^2}| + C$$

$$\therefore I = \frac{(x+2)}{2} \sqrt{x^2+4x-5} - \frac{9}{2} \log |(x+2) + \sqrt{x^2+4x-5}| + C$$

Question 7:

$$\sqrt{1+3x-x^2}$$

Answer

$$\begin{aligned}
 \text{Let } I &= \int \sqrt{1+3x-x^2} dx \\
 &= \int \sqrt{1-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)} dx \\
 &= \int \sqrt{\left(1+\frac{9}{4}\right)-\left(x-\frac{3}{2}\right)^2} dx \\
 &= \int \sqrt{\left(\frac{\sqrt{13}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2} dx
 \end{aligned}$$

It is known that, $\int \sqrt{a^2-x^2} dx = \frac{x}{2} \sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$

$$\begin{aligned}
 \therefore I &= \frac{x-\frac{3}{2}}{2} \sqrt{1+3x-x^2} + \frac{13}{4 \times 2} \sin^{-1} \left(\frac{x-\frac{3}{2}}{\frac{\sqrt{13}}{2}} \right) + C \\
 &= \frac{2x-3}{4} \sqrt{1+3x-x^2} + \frac{13}{8} \sin^{-1} \left(\frac{2x-3}{\sqrt{13}} \right) + C
 \end{aligned}$$

Question 8:

$$\sqrt{x^2+3x}$$

Answer

$$\begin{aligned}
 \text{Let } I &= \int \sqrt{x^2+3x} dx \\
 &= \int \sqrt{x^2+3x+\frac{9}{4}-\frac{9}{4}} dx \\
 &= \int \sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2} dx
 \end{aligned}$$

It is known that, $\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C$

$$\begin{aligned} \therefore I &= \frac{\left(x + \frac{3}{2}\right)}{2} \sqrt{x^2 + 3x} - \frac{9}{2} \log \left| \left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x} \right| + C \\ &= \frac{(2x+3)}{4} \sqrt{x^2 + 3x} - \frac{9}{8} \log \left| \left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x} \right| + C \end{aligned}$$

Question 9:

$$\sqrt{1 + \frac{x^2}{9}}$$

Answer

$$\text{Let } I = \int \sqrt{1 + \frac{x^2}{9}} dx = \frac{1}{3} \int \sqrt{9 + x^2} dx = \frac{1}{3} \int \sqrt{(3)^2 + x^2} dx$$

It is known that, $\int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + C$

$$\begin{aligned} \therefore I &= \frac{1}{3} \left[\frac{x}{2} \sqrt{x^2 + 9} + \frac{9}{2} \log |x + \sqrt{x^2 + 9}| \right] + C \\ &= \frac{x}{6} \sqrt{x^2 + 9} + \frac{3}{2} \log |x + \sqrt{x^2 + 9}| + C \end{aligned}$$

Question 10:

$$\int \sqrt{1+x^2} dx \text{ is equal to}$$

A. $\frac{x}{2} \sqrt{1+x^2} + \frac{1}{2} \log |x + \sqrt{1+x^2}| + C$

B. $\frac{2}{3} (1+x^2)^{\frac{2}{3}} + C$

C. $\frac{2}{3} x (1+x^2)^{\frac{3}{2}} + C$

$$\mathbf{D.} \quad \frac{x^2}{2}\sqrt{1+x^2} + \frac{1}{2}x^2 \log|x + \sqrt{1+x^2}| + C$$

Answer

$$\text{It is known that, } \int \sqrt{a^2 + x^2} dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2} \log|x + \sqrt{x^2 + a^2}| + C$$

$$\therefore \int \sqrt{1+x^2} dx = \frac{x}{2}\sqrt{1+x^2} + \frac{1}{2} \log|x + \sqrt{1+x^2}| + C$$

Hence, the correct Answer is A.

Question 11:

$$\int \sqrt{x^2 - 8x + 7} dx \text{ is equal to}$$

$$\mathbf{A.} \quad \frac{1}{2}(x-4)\sqrt{x^2 - 8x + 7} + 9 \log|x - 4 + \sqrt{x^2 - 8x + 7}| + C$$

$$\mathbf{B.} \quad \frac{1}{2}(x+4)\sqrt{x^2 - 8x + 7} + 9 \log|x + 4 + \sqrt{x^2 - 8x + 7}| + C$$

$$\mathbf{C.} \quad \frac{1}{2}(x-4)\sqrt{x^2 - 8x + 7} - 3\sqrt{2} \log|x - 4 + \sqrt{x^2 - 8x + 7}| + C$$

$$\mathbf{D.} \quad \frac{1}{2}(x-4)\sqrt{x^2 - 8x + 7} - \frac{9}{2} \log|x - 4 + \sqrt{x^2 - 8x + 7}| + C$$

Answer

$$\begin{aligned} \text{Let } I &= \int \sqrt{x^2 - 8x + 7} dx \\ &= \int \sqrt{(x^2 - 8x + 16) - 9} dx \\ &= \int \sqrt{(x-4)^2 - (3)^2} dx \end{aligned}$$

$$\text{It is known that, } \int \sqrt{x^2 - a^2} dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} \log|x + \sqrt{x^2 - a^2}| + C$$

$$\therefore I = \frac{(x-4)}{2} \sqrt{x^2 - 8x + 7} - \frac{9}{2} \log|(x-4) + \sqrt{x^2 - 8x + 7}| + C$$

Hence, the correct Answer is D.